Ramsey fringes formation during excitation of topological modes in a Bose-Einstein condensate

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Abstract

The Ramsey fringes formation during the excitation of topological coherent modes of a Bose-Einstein condensate by an external modulating field is considered. The Ramsey fringes appear when a series of pulses of the excitation field is applied. In both Rabi and Ramsey interrogations, there is a shift of the population maximum transfer due to the strong non-linearity present in the system. It is found that the Ramsey pattern itself retains information about the accumulated relative phase between both ground and excited coherent modes.

Key words: Bose-Einstein condensation, Ramsey fringes, Coherence, Non-linear dynamics
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1 Introduction

The creation of Bose-Einstein condensates (BEC) in non-ground states, as originally proposed by Yukalov, Yukalova, and Bagnato [1,2] is nowadays a topic of wide interest. This method allows for the direct formation of fragmented nonequilibrium condensates (see the review article by Leggett [3]).

One of important applications of coupling between different coherent modes of BEC is the possibility to produce various transverse modes in the atom laser [5,6,7,8]. Many experiments are devoted to the study of properties of trapped BECs by coupling their collective states [9,10].

Starting with a sample of atoms Bose-condensed in the ground state of a confining potential, it is possible to promote atoms from one trap level to another by using *resonance excitation* [1,11]. This is done by applying an additional weak external field, with a fixed spatial distribution, and oscillating in time with a frequency near the transition frequency between the ground and an excited state. Previous calculations demonstrated the possibility of macroscopic transfer between the levels, confirming the feasibility of this procedure, and analyzed several applications [1,2,11].

In the present work, we investigate the formation of the Ramsey-like fringes, due to the interference between the ground and a non-ground states of BEC, excited by means of a near resonant field. We keep in mind a time-domain version of the separated oscillatory-field method, as developed by Ramsey [13,14], consisting of a sequence of two Rabi $\pi/2$-pulses, which are equivalent to the oscillatory fields of the Ramsey method. Previously, the formation of Ramsey fringes in double Bose-Einstein condensates [15] was studied. But this is for the first time that the Ramsey patterns are obtained, when BECs states with different quantum numbers, associated with the trap potential, are coupled. The eventual measurement of those fringes would quantify the coherence of the process. An experimental setup for the observation of the topological coherent modes, through spatial distribution observation, is presently in progress in our research group [16,17].

This paper is structured as follows. In Sec. 2, we briefly review the dynamics of the coherent modes, based on the Gross-Pitaevskii equation for BEC [4]. Also, we recall the idea of the resonant excitation, which makes it possible to couple the ground and non-ground states. Section 3 is devoted to the formation of the Ramsey-like fringes and to the possibility of their experimental observation. In Section 4, we summarize our results.

2   Gross-Pitaevskii equation and coherent modes

At low temperatures, dilute Bose gas, as is known, is well described by the Gross-Pitaevskii equation. This equation describes the coherent states of the Bose system with the Hamiltonian

$$\hat{H} = \int \hat{\Psi}^\dagger (\mathbf{r}, t) \left[-\frac{\hbar^2 \nabla^2}{2m_0} + V_{\text{ho}} (\mathbf{r}, t) \right] \hat{\Psi} (\mathbf{r}, t) \, d\mathbf{r} +$$

$$+ \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \hat{\Psi}^\dagger (\mathbf{r}, t) \hat{\Psi}^\dagger (\mathbf{r}', t) V (\mathbf{r} - \mathbf{r}') \hat{\Psi} (\mathbf{r}', t) \hat{\Psi} (\mathbf{r}, t) , \tag{1}$$

with N bosons assumed to be confined by a harmonic trap potential, $V_{\text{ho}} (\mathbf{r}, t)$. The $\hat{\Psi} (\mathbf{r}, t)$ ($\hat{\Psi}^\dagger (\mathbf{r}, t)$) are the boson field operators that annihilate (create) an atom at position $\mathbf{r}$ and $V (\mathbf{r} - \mathbf{r}')$ is a two-body interaction due to atomic collisions. In a dilute cold gas, the most relevant scattering process is associated with elastic binary collisions at low energy [3,4], giving the effective interaction potential

$$V (\mathbf{r} - \mathbf{r}') = A_s \delta (\mathbf{r} - \mathbf{r}') = (N - 1) \frac{4\pi \hbar^2 a_s}{m_0} \delta (\mathbf{r} - \mathbf{r}') . \tag{2}$$

Here, a_s is the "zero-energy" s-*wave scattering length*. For an effective attractive (repulsive) interaction a_s is negative (positive). The confining harmonic potential is written as

$$V_{\text{ho}} (\mathbf{r}, t) = \frac{m_0}{2} \left(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2 \right) \tag{3}$$

where m_0 is the atomic mass and ω_k ($k = x, y, z$) are the trap oscillation frequencies along each axis.

The topological coherent modes are the solutions to the stationary Gross-Pitaevskii equation. The nonlinear coherent modes form an overcomplete basis and are normalized ($|\varphi_j|^2 = 1$). The eigenvalue problem for stationary functions is given by [11]

$$\hat{H} [\varphi_j] \varphi_j = E_j \varphi_j \tag{4}$$

where

$$\hat{H} [\varphi_j] = -\frac{\hbar^2 \nabla^2}{2m_0} + V_{\text{ho}} (\mathbf{r}, t) + A_s |\varphi_j|^2 \tag{5}$$

follows from the Hamiltonian (1).

The resonant pumping makes it possible a coherent transfer of condensed atoms between the collective levels of atoms in the harmonic trap, as is described in Ref. [11]. At initial time, N condensed atoms are assumed to be in the ground state of the trap with a frequency ω_0. The aim is to transfer the atoms to a non-ground state (labelled as p). The energy difference of the level-p, relative to the ground state, is $\hbar (\omega_p - \omega_0)$. Following Yukalov *et al.* [11], this transfer can be obtained through the action of an external oscillatory field given by

$$V_p (\mathbf{r}, t) = V (\mathbf{r}) \cos \omega t . \tag{6}$$

The corresponding non-linear Schrödinger equation, associated with the Hamil-

tonian (5), writes as

$$i\hbar\frac{\partial\phi\left(\mathbf{r},t\right)}{\partial t}=\left[\hat{H}+V_p\left(\mathbf{r},t\right)\right]\phi\left(\mathbf{r},t\right).\tag{7}$$

The modulating field is called external in the sense that it is not a part of the harmonic trap setup, but rather the field $V_p\left(\mathbf{r},t\right)$ is a perturbation in Eq.(7). The solution $\phi\left(\mathbf{r},t\right)$, can be expressed as a sum over the coherent modes: $\phi\left(\mathbf{r},t\right)=\sum_n c_n\left(t\right)\varphi_n\left(\mathbf{r},t\right)$. Although one could consider a more general form of the external field for transferring atoms between an arbitrary pair of collective levels [11], it is sufficient to work with the potential (6) in order to create the Ramsey interference fringes.

The functions $c_0\left(t\right)$, associated with ground state, and $c_p\left(t\right)$, ascribed to an excited state, determine the behavior of the populations in each mode

$$n_0\left(t\right)=\left|c_0\left(t\right)\right|^2\ ,\qquad n_p\left(t\right)=\left|c_p\left(t\right)\right|^2.\tag{8}$$

There are some conditions on the involved physical parameters in order to observe a macroscopic population of a non-ground state ($n_p\neq 0$): First, it is required that the resonance condition between the external field of frequency ω and the frequency associated with the level transition, $\omega_{p0}=\omega_p-\omega_0$, be fulfilled. Also, the detuning $\Delta\omega=\omega-\omega_{p0}$ should be small enough, $\left|\Delta\omega/\omega_{p0}\right|\ll 1$. At the same time, one should remember that not solely the external coupling filed, but also the interatomic collisions cause atomic transitions between the modes. In order to quantify both effects, it is convenient to define the interaction intensity α, associated with interatomic collisions, and the transition amplitude β of the coupling field as

$$\alpha_{m,k}=\frac{A_s}{\hbar}\int\left|\varphi_m\left(\mathbf{r}\right)\right|^2\left(2\left|\varphi_k\left(\mathbf{r}\right)\right|^2-\left|\varphi_m\left(\mathbf{r}\right)\right|^2\right)d\mathbf{r},$$
$$\beta=\frac{1}{\hbar}\int\varphi_0^*\left(\mathbf{r}\right)V_p\left(\mathbf{r}\right)\varphi_p\left(\mathbf{r}\right)d\mathbf{r}.\tag{9}$$

For simplicity, we set $\alpha\equiv\alpha_{0,p}=\alpha_{p,0}$ for the folowing analysis. These quantities have dimensions of frequency (Hz). Their values are to be smaller than the transition frequency ω_{p0}, so that

$$\left|\frac{\alpha}{\omega_{p0}}\right|\ll 1\ ,\qquad\left|\frac{\beta}{\omega_{p0}}\right|\ll 1.\tag{10}$$

For the time dependent process, with the potential V_p, the solution of Eq.(7) can be represented as

$$\phi\left(\mathbf{r},t\right)=\sum_n c_n\left(t\right)\varphi_n\left(\mathbf{r}\right)\exp^{-\frac{i}{\hbar}E_n t}.\tag{11}$$

4

Another condition is that the coefficients $c_n(t)$ evolve slowly in time, when compared to the oscillatory terms in the equation (11), so that

$$\frac{\hbar}{E_n}\left|\frac{dc_n}{dt}\right| \ll 1. \tag{12}$$

It is straightforward to obtain a system of coupled differential equations for the coefficients c_0 and c_p, whose detailed derivation can be found in Ref. [1],

$$i\frac{dc_0}{dt} = \alpha n_p c_0 + \frac{1}{2}\beta e^{i\Delta\omega t} c_p,$$
$$i\frac{dc_p}{dt} = \alpha n_0 c_p + \frac{1}{2}\beta^* e^{-i\Delta\omega t} c_0. \tag{13}$$

Let us mention that the form of these coupled equations resembles the equations of the Rabi two-level problem [18]. It is expected that, if we choose the appropriate physical parameters, Rabi oscillations between the topological modes could be observed. The solution to this system provides us with all necessary information about the dynamics of the populations, including both ground and non-ground states. An analytical solution of Eqs.(13) can be obtained when $|\beta| \ll |\alpha|$, as in Ref. [2]. Then the population fractions are given by

$$n_0 = 1 - \frac{|\beta|^2}{\Omega^2}\sin^2\frac{\Omega t}{2}$$
$$n_p = \frac{|\beta|^2}{\Omega^2}\sin^2\frac{\Omega t}{2}, \tag{14}$$

with the effective collective frequency Ω defined as

$$\Omega = \sqrt{|\beta|^2 + [\alpha(n_0 - n_p) - \Delta\omega]^2}. \tag{15}$$

It is worth emphasizing that this collective frequency is not a constant, but a function of the instant populations n_0 and n_p, and that the analytical solution is available under the condition that the populations of the modes have small changes. Thus, the population difference $\Delta n = |n_p - n_0|$ is almost a constant. When we are interested in a macroscopic population of an excited state, it is necessary to solve the coupled equations (13) using a numerical procedure based on the fourth-order Runge-Kutta method. Thus, we can deal with α and β values not restricted to the condition $|\beta| \ll |\alpha|$.

In order to find the time dependent solutions for coefficients $c_0(t)$ and $c_p(t)$, we consider as initial conditions $c_0(0) = 1$ and $c_p(0) = 0$ (all N atoms initially condensed in the ground state). In figure 1, we plot the results for the fractional populations, associated with the non-ground and excited states and
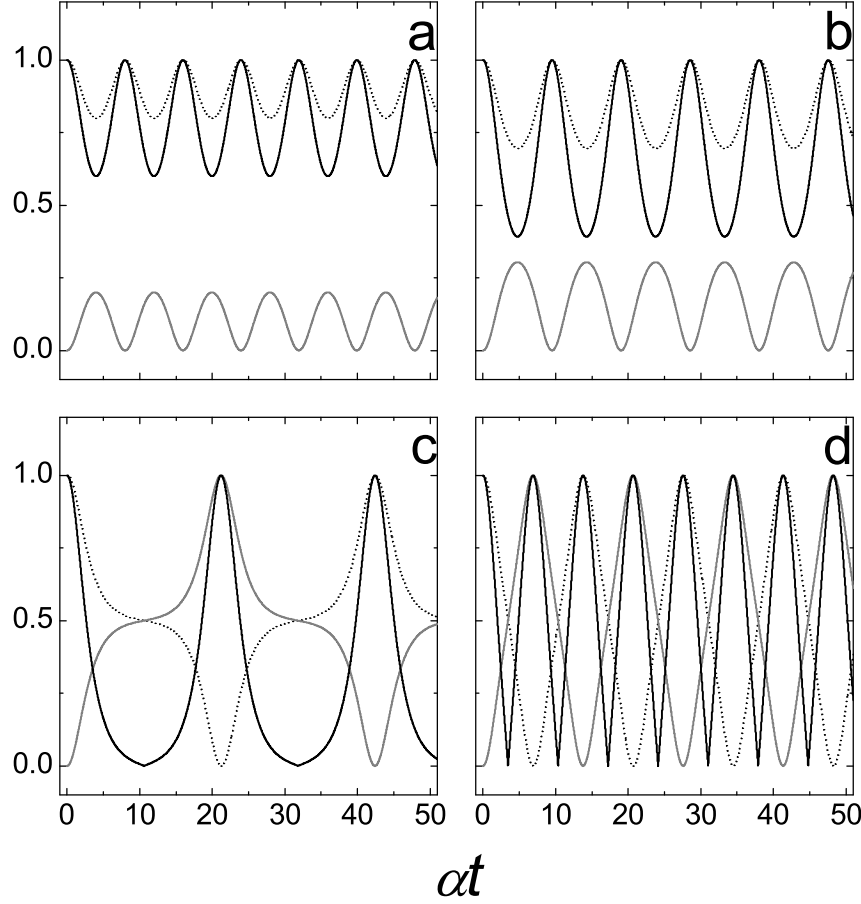
Fig. 1. Evolution of the density of populations (n_0 in dotted line, n_p in grey line) and the population unbalance $\Delta n = |n_p - n_0|$ (black line) as function of dimensionless quantity αt for $\Delta \omega \approx 0$ and different values of coupling parameter β: (a) $\beta = 0.40\alpha$; (b) $\beta = 0.46\alpha$; (c) $\beta \approx 0.50\alpha$ and (d) $\beta = 0.60\alpha$.

the population difference as functions of the dimensionless parameter αt. Grey lines show the population of the non-ground state ($n_p = 0$ at initial time) and dotted lines show $n_0(t)$. The evolution of the population difference, $\Delta n(t)$, is plotted in black bold lines. For all cases, the detuning is fixed so that $\Delta \omega \approx 0$, and we simulate a slight change of the external field V_p, which is characterized by the coupling parameter β defined in Eq.(9). From the previous works [19,11,2], we know that there are critical effects associated with the coupled system given by Eqs.(13), which, however, we shall not consider here.

Since it is possible to transfer populations between two different topological

6

states, under well defined conditions, prescribing when this process is efficient, we are in a position to analyze what happens if we manipulate the excitation time domain in order to detect Ramsey-like interference fringes as the oscillations of the fractional populations.

3 $\pi/2$ and π- pulses and the formation of the Ramsey-like fringes

In this section, we demonstrate the formation of the Ramsey-like fringes for the case of two topological modes coupled by an external resonant field. Given a certain value of β, we define the maximum of the atom population that is transferred from the ground to an excited state. It is also possible to estimate the necessary time for the coupling to be switched on. When a π pulse is applied, the time of the coupling, t_1, is sufficient for the performance of a complete Rabi oscillation. If the coupling is switched off at the time, when the half of the maximum population is transferred, the applied pulse is the so-called Rabi $\pi/2$-pulse. We show below that Ramsey fringes are obtained when two Rabi $\pi/2$-pulses are applied with a time interval τ between them. The latter procedure is similar to that one accomplished for the coupled hyperfine levels of Rubidium [20], and the formation of the fringes confirms the existence of a relative phase between both topological modes.

In figure 2, we plot our results for the fractional population density of the ground state, n_0, considering three pulse configurations, mentioned above, as functions of the detuning $\Delta\omega$. The normalized quantity n_0 gives us information about the atomic population in the mode and an indirect quantification of the visibility for the imaging process. The total time of the simulations is equal in all three cases. The procedure of obtaining the patterns implies the solution of system (13) for a fixed value of β, which means a fixed value of the spatial amplitude function in V_p, allowing for the variation of the detuning $\Delta\omega$.

In figure 2, black lines show the expected behavior of n_0, when equivalent Rabi pulses are applied. Both, π (bold line) and $\pi/2$ (thick line), pulses exhibit common features. First, we observe a sole peak, with a maximum at $\Delta\omega \neq 0$. The shift of the maximum of the resonant condition is associated with the contribution from the nonlinear terms of the Hamiltonian (5). Second, the maximual value of n_0 depends on the number of atoms transferred between the coupled states, which, in turn, depends on the coupling time t_1. This is related to the halfwidth $\Gamma \approx \frac{1}{t_1}$. If we compare both black lines in figure 2, we note that $\Gamma_\pi \sim 1.44$ and $\Gamma_{\pi/2} \sim 2.76 \approx 2\Gamma_\pi$.

The possibility of the formation of Ramsey fringes was first suggested in Ref. [11]. Our numerical calculations demonstrate that the fringes are really obtained when two $\pi/2$ pulses are applied separately. In this way, given a fixed
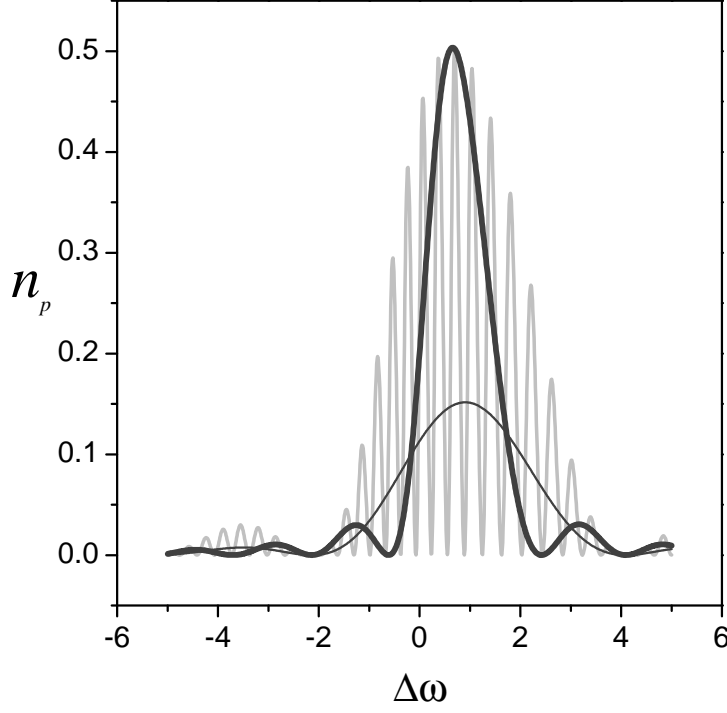
Fig. 2. The fractional population of a non-ground state, n_p, as a function of detuning $\Delta\omega$ for three different kinds of pulse configurations. For all cases $\beta = 0.4\alpha$. Grey line: Ramsey fringes due to the application of two $\pi/2$ pulses with an interval $\tau = 8t_1$; Bold black line: a π Rabi pulse; Thick black line: $\pi/2$ Rabi pulse.

number of condensed atoms N, approximately half of the population is transferred from the ground to an excited state during the first $\pi/2$ pulse, evolving freely when the coupling is switched off, and then, a second pulse concludes the excitation. The fractional population after this process, as a function of the detuning $\Delta\omega$, is plotted in figure 2 with the grey line. The process simulates the effect of the application of two $\pi/2$ Rabi pulses separated by $\tau = 8t_1$. The maximal value for n_p is found at the same frequency as in the configuration of a single pulse, which confirms our previous conclusion that the shift in the frequency is a hallmark of the nonlinear effects due to elastic two-body collisions.

In figure 3, we plot the behavior of the fractional population as a function of β, considering the same pulse configuration as above, with an initial $\pi/2$ pulse, an interval $\tau = 4t_1$, and a second $\pi/2$ pulse. If the coupling amplitude is weak, the Ramsey pattern loses its visibility. Increasing β improves visibility, but at the same time causes power shifts, represented by the fringes displacement. The number of the Ramsey fringes, contained within the spectral width, is strongly dependent on the time interval τ between the applied $\pi/2$- pulses. This feature is shown in figure 4, where four different situations are considered. We observe that the absolute maximal values of n_p and its position, as a function of $\Delta\omega$, are not connected with the changes of τ. What does depend
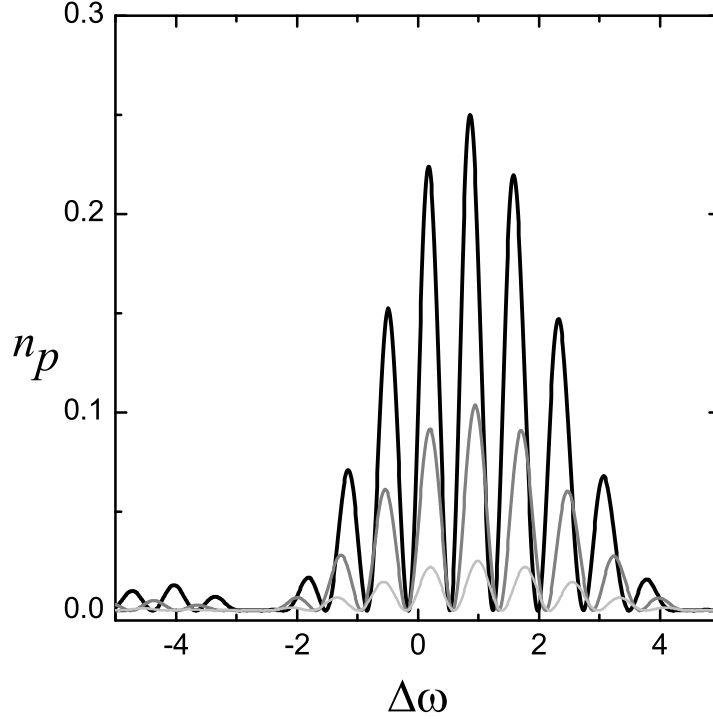
Fig. 3. Ramsey fringes of the fractional populatuion n_p as a function of $\Delta\omega$, after the application of two $\pi/2$- pulses separated by $\tau = 4t_1$, with t_1 being the time needed for the excitation of the half of the population from the ground to a non-ground state. The lines correspond to the increasing values of β. Light grey line: $\beta = 0.1\alpha$; Grey line: $\beta = 0.2\alpha$; Black line: $\beta = 0.3\alpha$

on τ is the number of fringes, which increases as τ increases. If τ is equal to the coupling time t_1, two additional peaks appear at the both sides of the main peak. For $\tau = 2t_1$, we obtain two peaks at each side, and so on, as it can be seen comparing the four plots in figure 4. The appearance of these auxiliary peaks is the hallmark for the accumulation of a relative phase between both topological states. During the time τ, when the coupling is switched off, the dynamics of the whole system (ground + non-ground state) is associated with the non-linear term in the Hamiltonian (5). This free dynamics determines the accumulated relative phase.
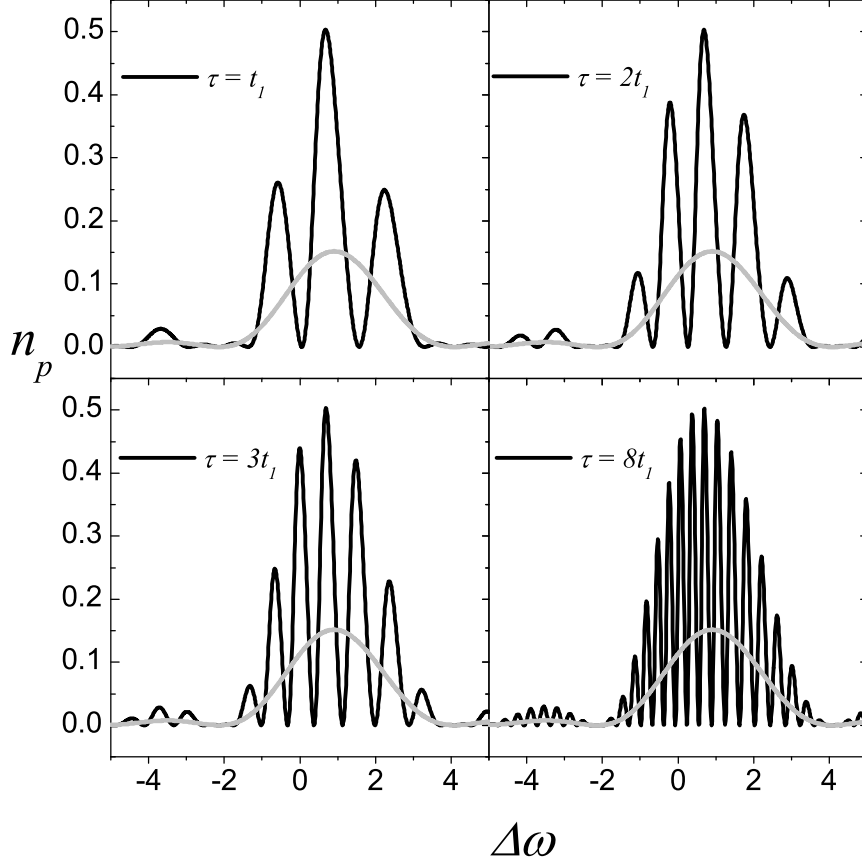
Fig. 4. Ramsey fringes of the fractional population n_p as a function of $\Delta\omega$, after the application of the Ramsey pulse configuration for different τ values. The gray line in all plots corresponds to the Rabi $\pi/2$-pulse.

4 Conclusions

In this work, we use numerical calculations for solving the system of coupled equations describing the resonant excitation of two coherent non-linear modes of BEC. Our results, obtained by means of the fourth-order Runge-Kutta method, give us an insight into the behavior of the fractional mode populations of non-ground collective atomic states in a harmonic trap. The principal novelty of the present paper is the investigation of the system response to different coupling configurations and the demonstration of the appearance of the Ramsey-like fringes.

The formation of the Ramsey-like fringes is a signature of the actual coherent

character of topological modes in the studied nonlinear system. In both, Rabi and Ramsey pulse configurations, there is a shift of the maximum population transfer due to the strong effect of non-linearity of the system. The Ramsey pattern itself contains information on the accumulated relative phase, and the number of secondary peaks is proportional to the time τ, as defined in Section 3.

As possible extension of this work, it would be interesting to consider the influence on the Ramsay fringes of the trap geometry and of different external pumping fields $V_p(\mathbf{r}, t)$. Other possibilities could be related to the manipilation of the scattering length *via* the Feshbach resonance techniques and to the effects of changing the total number of atoms, which affects the interatomic intensity α.

Acknowledgments

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